Techniques and Formulations for Mode Coupling of Multimode Optical Fibers

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Abstract—**Mode coupling, caused by random variations of core index or random irregularities of the fiber wall, influences the transmission characteristics of multimode optical fibers in a complicated way. The effects of mode coupling in multimode optical fibers have been reported using coupled power equations or power flow equations, and the good agreement between theoretical and experimental results provides further evidence that the power flow equation is a useful description of the power distribution in a multimode optical fiber. From these situations, it would be useful to develop general means to analyze power flow equations. In this paper, a means applicable to any coupling mechanism for treating the effects of mode coupling is given together with formal solutions.**

I. Introduction

PRESENTLY, multimode fibers are of interest in optical communication systems because of less stringent requirements of optical carriers and lower loss splices between fiber segments than single mode fibers. However, the dispersion caused by the delay differences among many modes of multimode fibers broadens optical signals, or limits the information-carrying capacity of the communication system. Moreover the mode coupling caused by the random variations of the core index or the random irregularities of the fiber wall complicates the characteristics of the pulse propagation.

Some papers [1]–[3] on the effects of mode coupling in multimode optical fibers have been reported. They have treated coupled power equations [4] or power flow equations [2] for some interesting but simple models. Koyama and Kobayashi [5] considered frequency characteristics of multimode fibers to be important in optical transmission system designs and proposed a perturbation method to analyze coupled power equations. They measured optical fiber transfer functions based upon the swept-frequency technique for baseband signals and verified the good agreement between theoretical and experimental results.

Mode coupling influences the transmission characteristics of multimode fibers in such a complicated way that it would be generally difficult to analyze those equations. The main purposes of this paper are to present the means to analyze power flow equations for any mode coupling mechanism, and to give formal solutions which can be evaluated numerically. As an example of one application, a coupling model for step-index fibers will be studied in detail, and the transmission characteristics are calculated.

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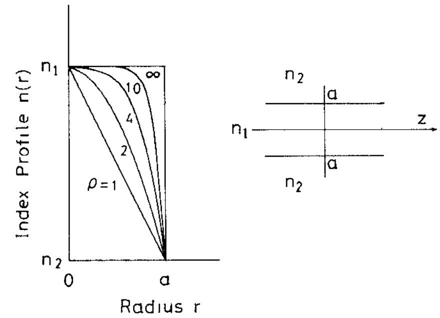

Fig. 1. The index profile of the core quoted from [6].

II. Power Flow Equation

To investigate the influences of the dispersion and the mode coupling in multimode fibers, the parabolic class of cylindrically symmetric-index profiles has been well studied [3], [6], [7].

$$n(r)=\begin{cases} n_1\left[1-2\Delta\left(\frac{r}{a}\right)^{\rho}\right]^{1/2}, & r<a \\ n_1(1-2\Delta)^{1/2}=n_2, & r>a \end{cases} \tag{1}$$

where n_1 is the maximum index of the core, n_2 is the index of the cladding, a is the core radius, and ρ characterizes the shape of the core index profile (Fig. 1). It is assumed that the relative index difference Δ is small compared to unity.

Each mode propagating in an optical fiber is specified by a pair of numbers (μ,ν) which are, respectively, the radial and azimuthal order numbers in the field intensity of that mode. For the step-index profile ($\rho=\infty$), approximate degeneracies exist among the propagation constants β of the guided modes, and the propagation constants depend only on a principal mode number m defined by

$$m=2\mu+\nu. \tag{2}$$

With this principal mode number, Gloge [2], who considered the mode coupling as a power diffusion, has derived a partial differential equation which describes the power flow of the coupled modes of step-index fibers:

$$\frac{\partial}{\partial z}P+i\omega\tau P=-\alpha P+\frac{1}{m}\frac{\partial}{\partial m}\left(mh\frac{\partial}{\partial m}P\right) \tag{3}$$

where P is the Fourier transform or the baseband response of the mode power, and the group delay τ, the loss constant α, and the power coupling coefficient h are functions of m which is treated as a quasi-continuous variable.

Olshansky [3] considered that it should be assumed that the degeneracies among the propagation constants also hold for the other values ρ because the degeneracies exist not only for the step-index profile ($\rho=\infty$) but also for the parabolic-index profile ($\rho=2$). These degeneracies have been derived using a scalar theory in Streifer and Kurtz [8], and using a vector theory in Kurtz and Streifer [9]. This paper also assumes that the power flow equation (3) describes the power flow of the coupled modes for any index profile ρ.

III. Application of Variational Techniques

The power flow equation (3) has been analyzed for some interesting but simple models [2], [3]. However, the mode coupling influences the transmission characteristcs of multimode fibers in a complicated way, so that it could not be generally represented by such simple models. From this situation, some general means to solve the power flow equation must be found. In this paper, variational techniques which can be applied to any index profile and to any mode coupling mechanism will be used to solve the power flow equation, and formal solutions will be represented.

The solution to (3) may be found by the technique of separation of the variables. Upon substituting a trial function

$$P(\omega,m,z)=e^{-\gamma(\omega)z}g(\omega,m) \tag{4}$$

with a parameter γ which is the function of the baseband frequency ω into (3), we obtain

$$\frac{1}{m}\frac{d}{dm}\left(mh\frac{d}{dm}g\right)+(\gamma-i\omega\tau-\alpha)g=0 \tag{5}$$

where partial derivatives $\partial/\partial m$ become total derivatives d/dm because g is a function only of the mode number m.

The boundary conditions [1] are as follows:

$$\left(\frac{d}{dm}g\right)_{m=m_0}=(g)_{m=M}=0 \tag{6}$$

where m_0 and M are, respectively, the minimum and maximum values of the mode number m. These boundary conditions show that no power can be lost at $m=m_0$ (the slope dg/dm determines the rate of the power diffusion) and no power propagates at the cutoff value $m=M$.

Next the variational representation for γ and g must be found. That is as follows:

$$\gamma(\omega,g)=\left[\int mh\left(\frac{d}{dm}g\right)^2 dm+i\omega\int m\tau g^2\,dm+\int m\alpha g^2\,dm -2\left(mhg\frac{d}{dm}g\right)_{m=M}\right]\Big/\int mg^2\,dm. \tag{7}$$

To determine whether this expression gives a solution for γ whose first variation is zero when the functional form of g is perturbed by a small amount δg, we calculate the variation.

$$\delta\gamma\int mg^2\,dm+2\gamma\int mg\delta g\,dm=2\int mhg'\delta g'\,dm +2i\omega\int m\tau g\delta g\,dm+2\int m\alpha g\delta g\,dm -2(mhg'\delta g)_{m=M}-2(mhg\delta g')_{m=M} \tag{8}$$

with

$$\int mhg'\delta g'\,dm=(mhg'\delta g)_{m=M}-(mhg'\delta g)_{m=m_0} -\int (nhg')'\delta g\,dm$$

where the symbol $'$ is used for the derivative for m, that is, d/dm.

The variation in γ is found to be

$$\delta\gamma\int mg^2\,dm=-2\int\left[(mhg')'+(\gamma-i\omega\tau-\alpha)mg\right]\delta g\,dm -2(mhg'\delta g)_{m=m_0}-2(mhg\delta g')_{m=M}. \tag{9}$$

Therefore, the solution of (5), subject to the boundary conditions (6), is equivalent to minimizing the functional γ independently of what boundary conditions δg and $\delta g'$ satisfy. As a trial function for g, a linear combination of $\{u_s(m),\ s\geq 1\}$ which is complete within the span (m_0,M) will be used, where each u_s satisfies the boundary conditions (6). Then g can be written as follows:

$$g(\omega,m)=\sum_{s\geq 1}a^s(\omega)u_s(m) \tag{10}$$

where a^s is the parameter to be determined.

Substituting (10) into (7) leads to

$$\gamma\int\sum_s\sum_l a^sa^l mu_su_l\,dm=\int\sum_s\sum_l a^sa^l mhu_s'u_l'\,dm +i\omega\int\sum_s\sum_l a^sa^l m\tau u_su_l\,dm+\int\sum_s\sum_l a^sa^l m\alpha u_su_l\,dm. \tag{11}$$

The conditions for minimizing γ may be written

$$\frac{\partial}{\partial\alpha^l}\gamma=0,\qquad l\geq 1$$

or substituting (11)

$$\gamma\int\sum_s a^s mu_su_l\,dm=\int\sum_s a^s mhu_s'u_l'\,dm +i\omega\int\sum_s a^s m\tau u_su_l\,dm+\int\sum_s a^s m\alpha u_su_l\,dm,\qquad l\geq 1. \tag{12}$$

These equations can be represented by the matrix form which is

$$(U+i\omega T+A)a(\omega)=\gamma(\omega)Sa(\omega) \tag{13}$$

where a is the column vector of a^s, and U, T, A, and S are symmetric matrices whose (s,l) elements are, respectively, defined by

$$U_{sl}=\int mhu_s'u_l'\,dm \tag{14}$$

$$T_{sl}=\int m\tau u_s u_l\,dm \tag{15}$$

$$A_{sl}=\int m\alpha u_s u_l\,dm \tag{16}$$

$$S_{sl}=\int m u_s u_l\,dm, \qquad s,l\geqslant 1. \tag{17}$$

The complete solution of (3) is given as the superposition of the trial functions.

$$P(\omega,m,z)=\sum_k C_k(\omega)\Big[\sum_s a_k^s(\omega)u_s(m)\Big]e^{-\gamma_k(\omega)z} \tag{18}$$

where the subscript k is used to label the eigenvalues γ and the eigenvectors a of (13). C_k should be determined from the initial condition at $z=0$.

Multiplying the both sides of (18) at $z=0$ by $m\Sigma_l a_r^l u_l$, and integrating over m lead to

$$\int m(P)_{z=0}\sum_l a_r^l u_l\,dm=\sum_k C_k\int m\Big(\sum_s a_k^s u_s\Big)\Big(\sum_l a_r^l u_l\Big)\,dm$$

$$=\sum_k C_k(a_k,Sa_r) \tag{19}$$

where (,) indicates a scalar product.

As is well known, the orthogonalities among the eigenvectors hold from (13) and the symmetries of the matrices U, T, A, and S.

$$(a_k,Sa_r)=0, \qquad k\neq r. \tag{20}$$

From (19) and (20), C_k is given by

$$C_k(\omega)=\frac{(a_k,b)}{(a_k,Sa_k)} \tag{21}$$

where b is the column vector whose lth element is defined by

$$b_l=\int m(P)_{z=0}u_l\,dm, \qquad l\geqslant 1. \tag{22}$$

In optical transmission systems, only the total power over all the modes is detected. Because mth group of the degenerate modes consists of about $2m$ modes [2], [3], the total power P_T can be written as follows:

$$P_T(\omega,z)\propto\int mP(\omega,m,z)\,dm=\sum_k C_k\Big(\int\sum_s a_k^s m u_s\,dm\Big)e^{-\gamma_k z}. \tag{23}$$

From (21) and (23), the total power P_T is given by

$$P_T(\omega,z)=\sum_k\frac{(a_k,b)(a_k,b_1)}{(a_k,Sa_k)}e^{-\gamma_k z} \tag{24}$$

where the proportional constant of (23) is set as unity for simplicity, and b_1 is the constant column vector whose elements are given by setting $(P)_{z=0}$ equal to unity in (22). That is

$$(b_1)_l=\int m u_l\,dm, \qquad l\geqslant 1. \tag{25}$$

Eigenvector given by Marcuse's algebraic eigenvalue equation [10] corresponds to the mode power directly, and the order of his matrix is equal to the number of the guided modes in a multimode fiber. Therefore, his eigenvalue equation is unsuitable for treating very many modes. Our eigenvalue equation is suitable for analyzing multimode fibers with many guided modes because the eigenvectors a_k determine not the mode power itself but the envelope which is the superposition of the mode power.

Transmission characteristics of multimode fibers are significantly influenced also by input mode power distribution [11]. For a coupling model first proposed by Gloge [2], [12], he has solved the power flow equation for the Gaussian input spatial distribution, and subsequently Gambling *et al.* [13] for a plane wave input and Cartledge [14] for a Lambertian input, respectively. Their derived solutions differ. Our method can treat not only any coupling mechanism as in previous works [1]–[3] but also any input mode distribution using (13)–(25), and obtain the final solution (24) with the same form.

IV. One Model of Coupling Mechanism

Once the forms of the parameters τ, α, and h are specified, the power flow equation can be solved by means of the variational techniques discussed above. As an example of an application, a step-index fiber will be studied. The group delay τ of the cylindrically symmetric index profile (1) has been derived by means of the WKB method [6].

$$\tau=\frac{n_1}{c}\left[1+\frac{\rho-2}{\rho+2}\Delta\Big(\frac{m}{M}\Big)^{2\rho/\rho+2}+\frac{1}{2}\,\frac{3\rho-2}{\rho+2}\Delta^2\Big(\frac{m}{M}\Big)^{4\rho/\rho+2}\right]. \tag{26}$$

For step-index fibers,

$$\tau=\frac{n_1}{c}\left[1+\Delta\Big(\frac{m}{M}\Big)^2\right], \qquad \rho=\infty. \tag{27}$$

The loss constant α and the power coupling coefficient h are specified by

$$\alpha=\alpha_0+\Delta_n\frac{AN}{L_s}\Big(\frac{m}{M}\Big)^2 \tag{28}$$

$$h=\frac{M^2}{L_s}\Big(\frac{m}{M}\Big)^2 \tag{29}$$

where

$$L_s\equiv\frac{a^2D}{\sqrt{\pi}\,\bar{\sigma}^2}. \tag{30}$$

See Fig. 2. α_0, Δ_n, A, and N are, respectively, the loss constant common to all the modes, the index difference n_1-n_2, a dimensionless constant and the total number of the guided modes. D and $\bar{\sigma}^2$ are, respectively, the correlation length and the variance of the random distortion of multimode fibers. These parameters are discussed in greater detail in the Appendix.

Substituting (27)–(29) into (3), we obtain

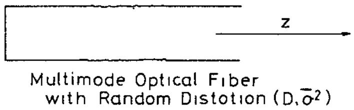

Fig. 2. The multimode optical fiber with the random distortion whose correlation length and variance are, respectively, D and $\bar{\sigma}^2$.

$$\frac{\partial}{\partial z}P+\left(\alpha_0+i\omega\frac{n_1}{c}\right)P=-\Delta_n\left(AN+i\omega\frac{L_s}{c}\right)\frac{x^2}{L_s}P+\frac{1}{x}\frac{\partial}{\partial x}\left(\frac{x^3}{L_s}\frac{\partial}{\partial x}P\right) \quad (31)$$

where

$$x\equiv\frac{m}{M}, \qquad 0\leqslant x\leqslant 1. \quad (32)$$

In this model, it is convenient to consider the next trial function for P

$$P(\omega,x,z)=\exp\left[-(\alpha_0+i\omega n_1/c)z\right]g(\omega,x)\cdot\exp\left[-\gamma(\omega)z/L_s\right]. \quad (33)$$

Following procedures similar to those of §.3, the mode power P and the total power P_T, which are slightly modified from (18) and (24), become as follows:

$$P(\omega,x,z)=\exp\left[-(\alpha_0+i\omega n_1/c)z\right]\sum_k\frac{(a_k,b)}{(a_k,Sa_k)}\cdot\left[\sum_s a_k^s u_s(x)\right]\exp\left[-\gamma_k z/L_s\right] \quad (34)$$

$$P_T(\omega,z)=\exp\left[-(\alpha_0+i\omega n_1/c)z\right]\sum_k\frac{(a_k,b)(a_k,b_1)}{(a_k,Sa_k)}\cdot\exp\left[-\gamma_k z/L_s\right] \quad (35)$$

where the function $u_s(x)$ is defined within the span (0, 1), and b and b_1 are, respectively, given from (22) and (25) by replacing m by x, and γ_k and a_k are the eigenvalue and the eigenvector of

$$(U+\theta T)a_k(\omega)=\gamma_k(\omega)Sa_k(\omega), \qquad k\geqslant 1 \quad (36)$$

with

$$\theta\equiv\Delta_n\left(AN+i\omega\frac{L_s}{c}\right). \quad (37)$$

The matrix θT of this model corresponds to the general form $i\omega T+A$ of (13), and the elements of U, T, and S are defined by

$$U_{sl}=\int x^3u_s'u_l'\,dx \quad (38)$$

$$T_{sl}=\int x^3u_su_l\,dx \quad (39)$$

$$S_{sl}=\int xu_su_l\,dx, \qquad s,l\geqslant 1. \quad (40)$$

V. Transmission Characteristics of Multimode Fiber for the Above Model

After the transient state has died out, the propagating pulse approaches a Gaussian provided the pulse spread is such that its width is much larger than the width of the input pulse of arbitrary shape, and the steady state distribution of the power over all the modes has become independent of the initial condition [15]. Since the steady-state power flow is most important, it is sufficient to consider the input pulse as an impulse both in time and in space, that is $(P)_{z=0}=1$. However, input pulses of any shape can be treated similarly.

A. Power Distribution for CW Excitation

The power distribution for continuous wave excitation, which corresponds to the baseband frequency $\omega=0$, will be studied with the above model. The power loss that results from the coupling of the guided modes directly to the radiation field is ignored, that is A of (28) = 0.

1) Attenuation of Mode Power: For $\omega=0$, the mode power becomes as follows:

$$P(0,x,z)=\sum_k\frac{(a_{0k},b_1)}{(a_{0k},Sa_{0k})}\left[\sum_s a_{0k}^s u_s(x)\right]\exp\left[-\gamma_{0k}z/L_s\right] \quad (41)$$

where the term $\exp(-\alpha_0 z)$ common to all the modes is ignored in the remainder of this paper because this loss can be accounted for later by multiplying the final solutions, and b becomes b_1 for $(P)_{z=0}=1$ from (22) and (25). γ_{0k} and a_{0k} are, respectively, the eigenvalue and the eigenvector of

$$Ua_{0k}=\gamma_{0k}Sa_{0k}, \qquad k\geqslant 1. \quad (42)$$

In the steady state ($z/L_s\gg 1$), the mode power distribution becomes independent of the initial condition at $z=0$.

$$P(0,x,z)\propto\sum_s a_{01}^s u_s(x)\exp\left[-\gamma_{01}z/L_s\right], \qquad z/L_s\gg 1 \quad (43)$$

where γ_{01} is the minimum eigenvalue of (42), and this relation holds for any initial excitation by replacing γ_{01} and a_{01}, respectively, by γ_1 (whose real part is the smallest of those of the eigenvalues of (36)) and a_1 of (36). In Fig. 3, attenuation ratio is defined by the relative mode power to the fundamental mode power, that is, $P(0,x,z)/P(0,0,z)$. Fig. 3 shows a mode filtering effect due to differential mode attenuation [16]: that is, the higher mode suffers a relatively higher loss compared to that of the lower mode.

2) Optical Power Flow: The mode power P is not the real power flow because the number of the modes specified by m are approximately $2m$. Therefore, the true power flow specified by m is proportional to mP or xP, which is called the optical power flow in this paper. For step-index fibers, the output angle of the power flow from the fiber end is proportional to the mode number m,

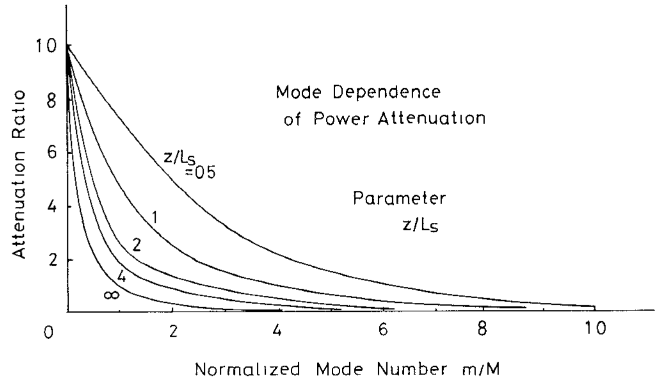

Fig. 3. The attenuation of the mode power versus the normalized mode number $m/M = x$ on some parameters of the normalized fiber length z/L_s when all the modes equally excited at the input $z=0$, where the fundamental mode power is normalized to unity and $z/L_s = \infty$ corresponds to the steady state.

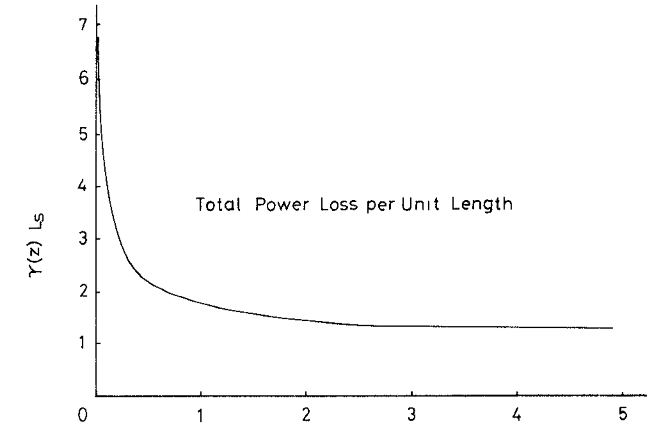

Fig. 5. The power loss per unit length.

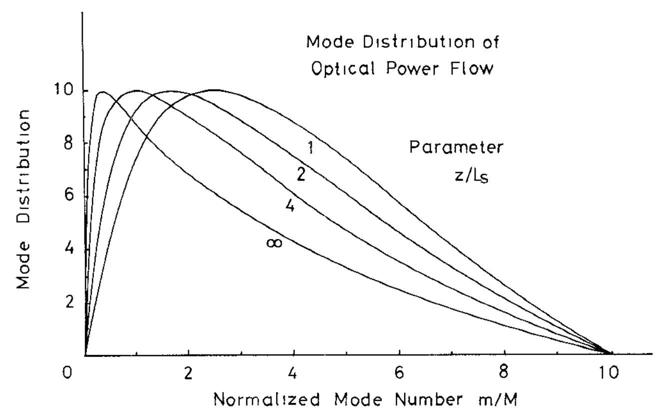

Fig. 4. The optical power flow which is normalized so that the peak values are unity.

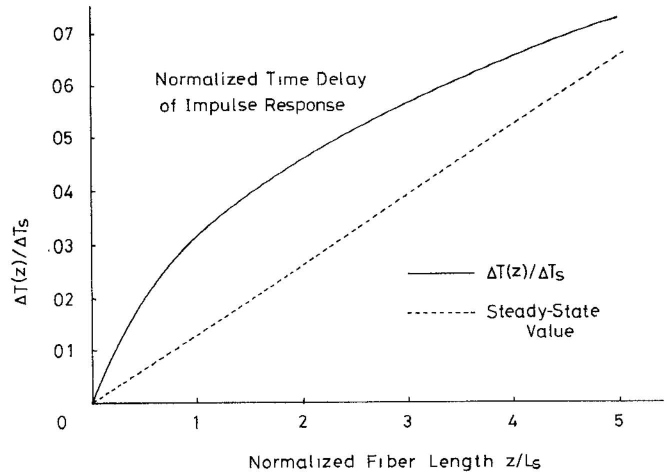

Fig. 6. The upper line is the delay time $\Delta T(z)$ normalized by ΔT_s ($=\Delta_n L_s/c$ is the width of the distorted pulse at the fiber length $z = L_s$ in the absence of the mode coupling) and the lower line corresponds to the steady state values.

therefore the optical power flow versus the mode number corresponds to the output pattern from the fiber end [2]. Mode distribution in Fig. 4 is the optical power flow normalized so that the peak value is unity at given fiber length z. As shown in Fig. 4, the effective spatial bandwidth or the effective numerical aperture decreases to some fiber length and eventually becomes a constant value.

3) Loss of the Total Power: Since the coupling among the guided modes must also cause the coupling of the guided modes to the radiation field, the radiation loss is unavoidable, and it is therefore important to investigate the radiation loss. The power loss per unit length is

$$\gamma(z) = -\frac{1}{P_T(0,z)} \frac{d}{dz} P_T(0,z) \tag{44}$$

$$P_T(0,z) = \sum_s \frac{(a_{0k}, b_1)^2}{(a_{0k}, Sa_{0k})} \exp\left[-\gamma_{0k} z/L_s\right]. \tag{45}$$

Attenuation of a multimode fibers with the mode coupling varies with fiber length because of mode selective attenuation. However, as shown in Fig. 5, the power loss per unit length approaches to a constant value describing its steady-state value as the optical power propagates in a multimode fiber.

B. Delay Time and Spread of Impulse Response

The transmission characteristics of optical fibers can be described by specifying the moment $M_n(z)$ of the full impulse response. This moment is defined by

$$M_n(z) = \begin{cases} \int_{-\infty}^{\infty} t^n P_T(t,z)\, dt \\ (i)^n \dfrac{\partial^n}{\partial \omega^n} P_T(\omega,z)_{\omega=0} \end{cases}, \quad n \geqslant 0 \tag{46}$$

where $P_T(t,z)$ is the inverse Fourier transform of $P_T(\omega,z)$.

1) Delay Time of Impulse Response: The delay time $T(z)$ of the impulse response is given by (see Fig. 6)

$$T(z) \equiv \frac{n_1}{c} z + \Delta T(z) = \frac{M_1(z)}{M_0(z)}. \tag{47}$$

Continuous mode mixing reduces the signal distortion, and eventually forces all the modes to propagate at an average velocity.

2) Spread of Impulse Response: The rms width $\sigma(z)$ of the impulse response is given by (see Fig. 7)

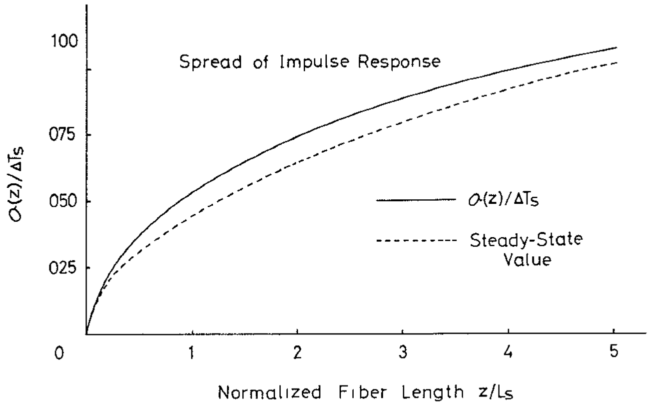

Fig. 7. The upper line is the rms width $\sigma(z)$ normalized by ΔT_s and the lower line is the rms width $\sigma_s(z)$ in the steady state.

$$\sigma(z)=\left[\frac{M_2(z)}{M_0(z)}-T(z)^2\right]^{1/2}. \tag{48}$$

It is well known that the modal delay distortion will increase linearly up to some fiber length (the transient state) and thereafter increase as the square root of length (the steady state).

3) Delay Time and Spread of Impulse Response in Steady State: Next we proceed to describe the approximate evaluation of the eigenvalue equation (36) by perturbation techniques. For small value of θT, the solutions of (36) are only slightly different from those of (42). For our purposes, the second-order perturbations of the eigenvalues are most important. From second-order perturbation theory, the eigenvalues can be approximately written as follows:

$$\gamma_k(\omega)=\gamma_{0k}+\theta X_k+\theta^2 Y_k, \qquad k\geqslant 1 \tag{49}$$

with

$$X_k=\frac{(a_{0k},Ta_{0k})}{(a_{0k},Sa_{0k})} \tag{50}$$

$$Y_k=\frac{1}{(a_{0k},Sa_{0k})}\sum_{r\neq k}\frac{(a_{0r},Ta_{0k})^2}{(\gamma_{0k}-\gamma_{0r})(a_{0r},Sa_{0r})}. \tag{51}$$

Only the first term of the series of (35) needs to be considered for large z if $\gamma_{01}>\gamma_{0k}$ for all $k\geqslant 2$. The steady-state impulse response P_s becomes

$$P_T(\omega,z)\rightarrow P_s(\omega,z)=\frac{(a_1,b_1)^2}{(a_1,Sa_1)}\exp\left[-\gamma_1 z/L_s\right], \qquad z/L_s\gg 1. \tag{52}$$

Now the rms width $\sigma_s(z)$ and the delay time $\Delta T_s(z)$ in the steady state can be determined from (37), (42), and (49)–(51).

$$\sigma_s(z)=\Delta T_s\left(2Y_1\frac{z}{L_s}\right)^{1/2} \tag{53}$$

$$\Delta T_s(z)=\Delta T_s(X_1-2\Delta_n ANY_1)\frac{z}{L_s} \tag{54}$$

where

$$\Delta T_s\equiv\Delta_n\frac{L_s}{c}. \tag{55}$$

In Figs. 6 and 7, the lower lines are, respectively, $\Delta T_s(z)$ and $\sigma_s(z)$.

As the fiber length becomes large, $\Delta T(z)$ and $\sigma(z)$ approach $\Delta T_s(z)$ and $\sigma_s(z)$, respectively. These circumstances show that all the power of the guided modes propagates approximately at the same velocity, and the impulse response broadens as the square root of the fiber length in the steady state.

Equation (53) can be rewritten as follows:

$$\sigma_s(z)=\Delta_n\frac{z}{c}\left(2Y_1\frac{L_s}{z}\right)^{1/2}. \tag{56}$$

$\Delta_n z/c$ is the width of the distorted pulse at the end of a fiber carried by all the modes in the absence of the mode coupling [6], therefore the form of (56) corresponds to the result that Marcuse [17] has predicted. L_s of (30) may be considered as a characteristic that is required for the steady state to establish itself.

VI. Conclusion

A method of analyzing power flow equations were investigated, and the formal solutions of these equations were represented. This method can be applied to any coupling mechanism, and also to different types of partial differential equations provided the mode number is assumed to be continuous. As an example of applications, some characteristics of multimode fibers were calculated for a coupling model on step-index fibers. Frequency characteristics of multimode fibers are important to design SNR, equalizers, etc. in an optical communication system. The method developed in this paper is useful especially to relate frequency characteristics and mode coupling effects in multimode fibers.

VII. Appendix

The power coupling coefficient h is given by Marcuse [4].

$$h=|K_{m,m\pm 1}|^2 F(\beta_m-\beta_{m\pm 1}) \tag{57}$$

where $K_{m,m\pm 1}$ is the mode coupling coefficient between the mode m and $m\pm 1$, and $F(\beta_m-\beta_{m\pm 1})$ is the power spectrum of the coupling function $f(z)$ which describes the random distortion of multimode fibers [4].

If the coupling function $f(z)$ is supposed to be a random stationary variable whose correlation function is assumed to be a Gaussian, the power spectrum becomes

$$F(\beta_m-\beta_{m\pm 1})=\sqrt{\pi}\,\bar{\sigma}^2 D e^{-(D/2)^2(\beta_m-\beta_{m\pm 1})^2}. \tag{58}$$

For purposes of multimode operation, approximate expressions hold for the propagation constants and the coupling coefficients which are valid far from cutoff [16].

$$\beta_m-\beta_{m\pm 1}=\frac{1}{2n_1 K}(\kappa_m^2-\kappa_{m\pm 1}^2) \tag{59}$$

$$\kappa_m = \frac{m\pi}{2a} \tag{60}$$

$$K_{m,m\pm 1} = \frac{\kappa_m \kappa_{m\pm 1}}{2in_1 K} \tag{61}$$

where K is the wave number of free space and κ_m is the radial component of K.

From (57)–(61), the power coupling coefficient h becomes (29) of §.4 with the assumption

$$\exp\left[-(D/2)^2(\beta_m - \beta_{m\pm 1})^2\right] = \frac{(2/D)^2}{(\beta_m - \beta_{m\pm 1})^2},$$

$$(2/D)^2 \ll (\beta_m - \beta_{m\pm 1})^2. \tag{62}$$

The loss caused by coupling directly to the radiation field becomes [7]

$$\alpha = \sum_\mu \int_{-n_2K}^{n_2K} |K_{m,\mu}|^2 F(\beta - \beta_m) \frac{|\beta|}{\rho} \, d\beta \tag{63}$$

with

$$\rho = \sqrt{n_2 K^2 - \beta^2}$$

where Σ_μ means the summation for the different types of the radiation fields.

For most scattering problems, it is sufficient to use the coupling coefficients to the radiation fields of free space. In this case [16],

$$\alpha = \Delta_n \frac{\kappa_m^2 \sqrt{\pi}\, \bar{\sigma}^2}{2a^2 D} A \tag{64}$$

with

$$A = \sum_\mu \frac{e_{\nu\mu 1}^2}{e_\nu e_\mu} \int \frac{n_2 K D^2 J_\mu^2(\rho a)}{(\beta^2 + n_2^2 K^2)} \exp\left[-(D/2)^2(\beta - \beta_m)^2\right] d\beta \tag{65}$$

where the factors e_ν, e_μ, and $e_{\nu\mu 1}$ are specified by Marcuse [16].

From (60) and $N = \pi^2 M^2/8$, the power loss α becomes (28) of §.4, which loss includes the power loss α_0 in the absence of the mode coupling. A defined by (65) becomes independent of the distortion provided $(2/D)^2 \ll (\beta - \beta_m)^2$.

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